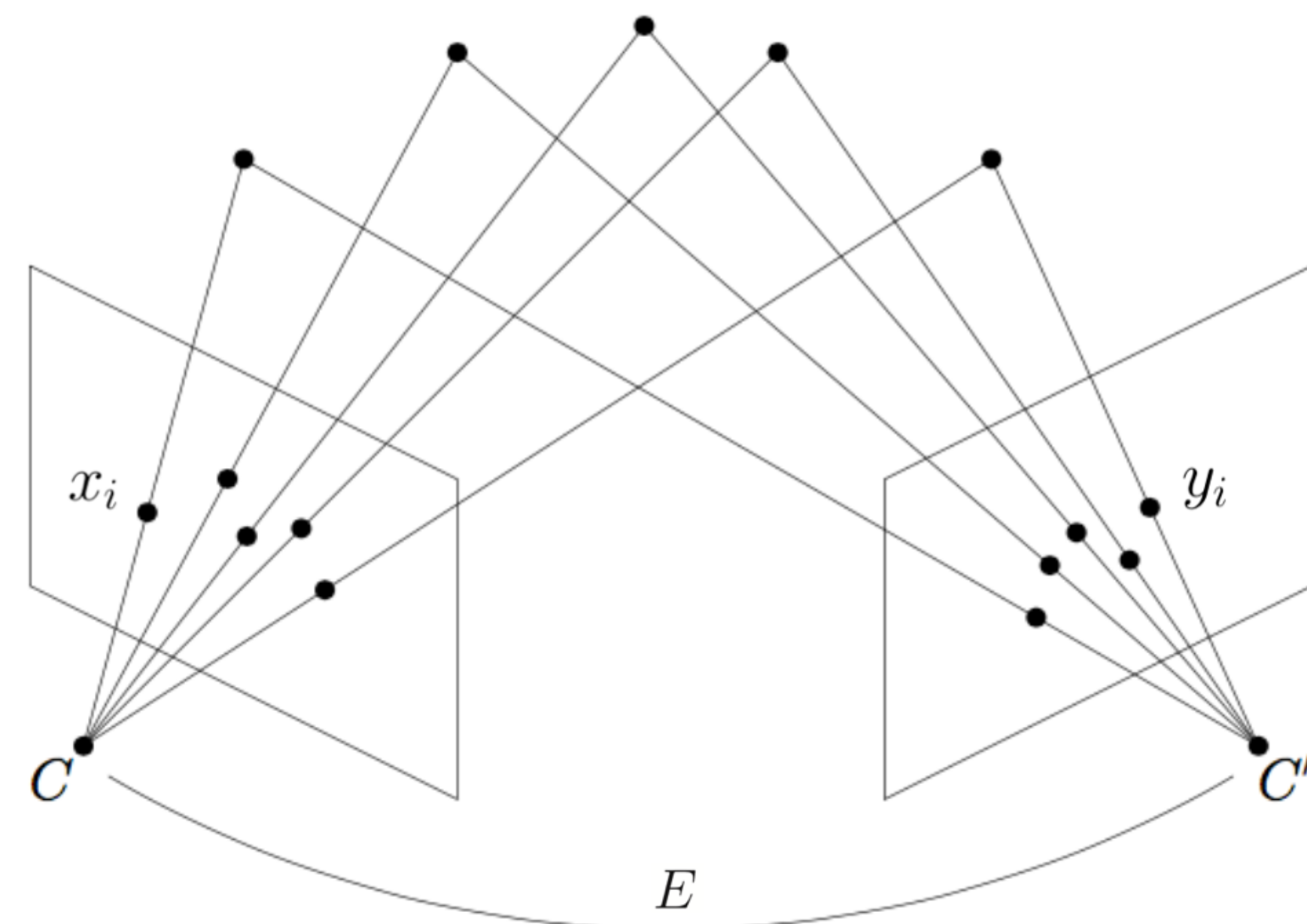


Overview

Minimal problems in computer vision involve systems of polynomial equations that have a finite number of solutions.



- 5-point problem³:
 - Minimal problem with 10 solutions.
 - Two cameras have corresponding data, x_i and y_i , arising from 5 points on an object in space.
 - Reconstruct relative position, E , of two cameras.
 - Polynomial system is:

$$\begin{bmatrix} 2EE'E - \text{trace}(EE')E \\ y_i'Ex_i \end{bmatrix} \begin{matrix} \leftarrow 9 \text{ polynomials} \\ \leftarrow 5 \text{ polynomials} \end{matrix} \text{ for } i = 1 : 5$$
 - Overdetermined system with 14 polynomials on $\mathbb{P}^8$!

Future Work



- Larger systems in computer vision.
 - 6-point problem with radial distortion
- Software development - joint with S. Agarwal (Google).

Goals

To use homotopy continuation methods to solve minimal problems that:

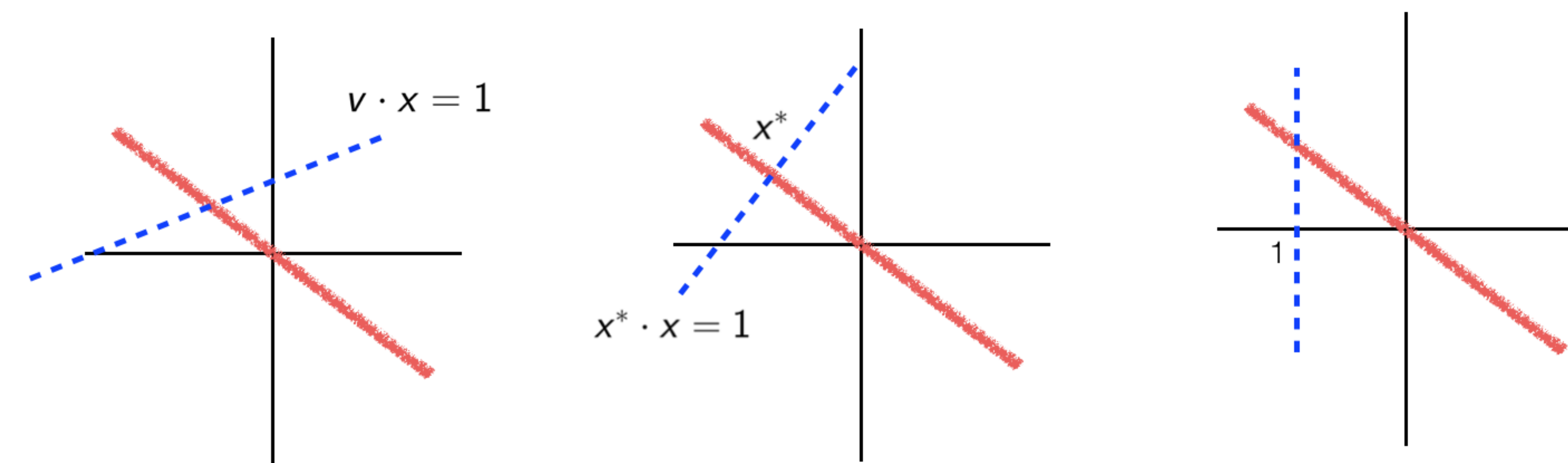
- Resolve numerical instability that arises from having an overdetermined system by randomizing to create a square system.
- Increase efficiency when moving from projective to affine space.

Randomization Strategies

$$R \cdot f(x)$$

- (Global) Fixed Randomization - choose R randomly.
- (Adaptive) Pseudoinverse Randomization - $R = [\text{inv}(\Sigma) \cdot U']$, from SVD of Jacobian.
- (Adaptive) Leverage Score Randomization - R = permuted diagonal matrix based on leverage scores².

Patching Strategies



Fixed Patching

Orthogonal Patching

Coordinate-Wise Patching

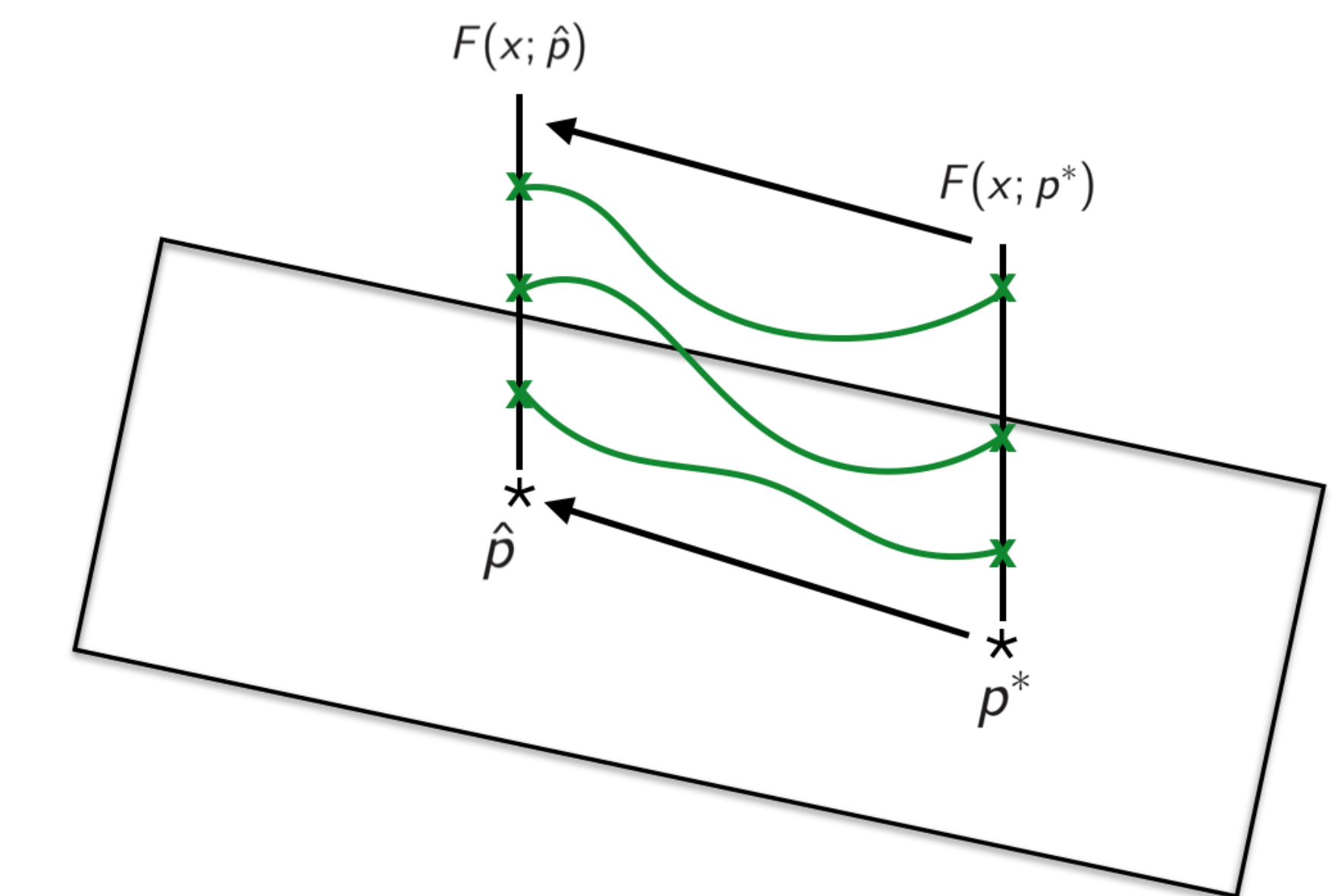
- Main idea: replace computations on lines (projective space) with points (affine space).
- (Global) Fixed Affine Patch⁴ - choose v randomly, $v \in \mathbb{C}^{N+1}$, such that $v \cdot x = 1$.
- (Adaptive) Orthogonal Affine Patch⁶ - take $x^* \cdot x = 1$, where $x^* \in \mathbb{C}^{N+1}$, with $\|x^*\|_2 = 1$.
- (Adaptive) Coordinate-Wise Affine Patch - take $e_j \cdot x = 1$, where $j = \arg \max_i |x_i^*|$.

Results

	Baseline	New Method
Avg. # Iterations	14.75	8.3

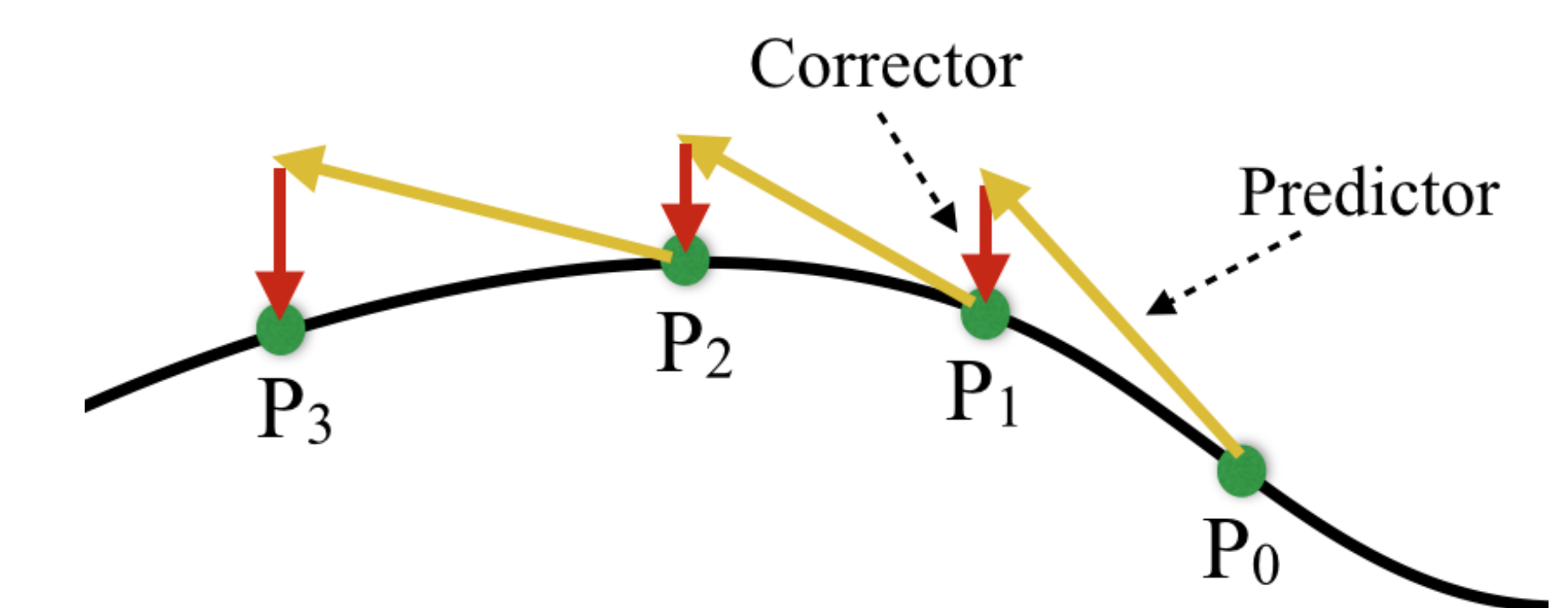
- About 40% decrease in average number of iterations from globally fixed algorithm to coordinate-wise patching and leverage score randomization algorithm with early truncation for the 5-point problem.
- Results are averaged from 100 random runs in Matlab.

Parameter Homotopy⁵



- Defined by $H(x, t) = F(x; p^*t + (1 - t)\hat{p})$, where:
 - $F(x; \hat{p})$: target system
 - $F(x; p^*)$: start system

Path Tracking¹



- "Predictor" step to extrapolate next point given previous data - Classical 4th Order Runge Kutta.
- "Corrector" step to refine the predicted value - maximum of 3 Newton iterations.

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